# Effects of AP Particle Size on Combustion Response to Cross Flow

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**An analytical model is developed for the linearized velocity-coupled combustion response function. The model treats elements of response to perturbations in pressure, in composition due to the heterogeneity of composite propellants, and in cross-flow velocity. The effects of AP particle size are accounted for in terms of effects on controlling ballistics properties and in terms of fluctuations in propellant composition. There are two facets of the cross-flow problem: the effect of cross-flow velocity on the various response elements, and the response to velocity perturbations. Both are dealt with in this paper. Important trends derived from series of parametric computations are described.**

## Nomenclature

| | |
|---|---|
| $a$ | = speed of sound |
| $D$ | = AP particle size |
| $E$ | = erosive transport defined by Eq. (1) |
| $K_1,K_2$ | = constants |
| $K_\rho$ | = sensitivity of propellant density to AP concentration |
| $L$ | = common denominator in response function expressions, defined by Eq. (7) |
| $m$ | = mass flux |
| $N$ | = exponents in Eqs. (1) and (2) |
| $n$ | = exponents pertaining to response function elements, Eqs. (4-6) |
| $P$ | = pressure |
| $r_0$ | = mean burn rate in the presence of cross flow |
| $r_i$ | = mean burn rate in the absence of cross flow |
| $R_c$ | = classical pressure-coupled response function component,[1] Eq. (4) |
| $R\alpha$ | = compositional response function component,[1] Eq. (5) |
| $R_u$ | = velocity-coupled response function component, Eq. (6) |
| $T_w$ | = surface temperature |
| $T_i$ | = initial propellant temperature |
| $u$ | = cross-flow speed |
| $u_t$ | = threshold cross-flow speed |

*Subscripts*

| | |
|---|---|
| $0$ | = mean value (subscript) |
| $\alpha$ | = AP concentration |
| $\epsilon$ | = error defined by Eq. (3) |
| $\lambda$ | = complex quantity characterizing the time dependent solution for the condensed phase |
| $\Omega$ | = dimensionless frequency |
| $\sigma_p$ | = temperature sensitivity of burn rate[1] |
| $\sigma_T$ | = temperature sensitivity of surface temperature [1] |

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## Introduction

THIS paper is a sequel to Ref. 1, which is concerned with effects of ammonium perchlorate (AP) particle size on the combustion response of composite solid propellants to perturbations in composition and pressure. The response to compositional fluctuations is a mechanism by which the heterogeneity of composite propellants can contribute directly to the combustion driving of instability. The compositional fluctuations originate in the solid phase, due to the heterogeneity and the passage of the burning front. These may couple with time lags in response to pressure and velocity perturbations. This paper shows the effects on combustion response to perturbations in cross-flow velocity or linear velocity-coupling. While there are no systematic data on the effects of AP particle size on the velocity-coupled response function, experiences with triggered nonlinear instability—in which velocity-coupling is believed to be a major factor—suggest that particle size is quite important.[2-6]

There are two facets of the response to cross flow. First, as pointed out by Lengelle, is the effect of cross flow on the various response function elements—including pressure coupling and various steady-state parameters that enter into the calculations in accordance with classical theory. Second is the combustion response to perturbations in the cross flow, which is the response function of interest here. Both are dealt with in this paper. The analysis is limited to conditions under which quasisteady gas assumptions (for the wall layer as well as the combustion) and principles of linearization apply.

## Heat Feedback Law

The effect of cross flow basically involves a boundary layer-type heat transfer mechanism, by which the heat feedback from the combustion zone is augmented. Under steady-state conditions, the phenomenon is commonly referred to as "erosive burning." Where the flow has an oscillatory component, the combustion response to these oscillations is sometimes referred to as the "erosive response," but is more often called the "velocity-coupled response." A number of theoretical analyses describing the way that cross flow increases the heat feedback to the burning surface have been published in recent years.[7-16] There are differences and disagreements in describing the mechanism which have not been resolved, even for steady-state burning.

It was not the intention of this work to add to the literature on increased heat feedback mechanisms, but rather to concentrate on the combustion response (with emphasis on AP particle size effects) to whatever the perturbing function is that is transmitted. Therefore, a semi-empirical approach was adopted to relate the change in heat feedback to cross flow.

The relation between the heat feedback and oscillatory flow is a subject of continuing work by others, and can be incorporated when sufficient progress is achieved.

It was assumed that the influence of cross flow could be represented by an addition to the surface energy balance equations taking the form of "Saderholm's Law." Starting with the Cohen and Strand combustion model,[17] the equations for the surface temperatures of AP and binder were modified by adding an erosive term, $E$, of the form

$$E = K_1 (u_0/u_t)^{N_u} (P_0) N_p \tag{1}$$

This particular law was selected because it has a good history of correlating erosive burning data,[5,18,19] appears to have the proper interplay between controlling variables that a more scientific theory would have to match, and has been given some mechanistic support in the work of Razdan and Kuo.[14,20] Assuming the erosive threshold to increase with increasing burn rate and decreasing AP particle size[9] (there is also a pressure effect[9]), Eq. (1) becomes

$$E = K_2 (u_0)^{N_u} (P_0)^{N_p} (D)^{N_D} / (r_0)^{N_r} \tag{2}$$

Equation (2) was quantified by fitting the Cohen and Strand model, as modified, to King's[13] experimental data. As optimized, the resulting standard deviation between calculated and experimental burning rates was 24% when compared as

$$\epsilon = \frac{|(r_0/r_i)_{\text{calc.}} - (r_0/r_i)_{\text{exp.}}|}{(r_0/r_i)_{\text{exp}} - 1} \times 100\% \tag{3}$$

This is quite good because it compares the incremental burning rates rather than the burning rates themselves. Also, it better reflects the error between the calculated value of $E$ and that value of $E$ required in the model to match the experimental result exactly.

It is of interest that the particle size dependence, $N_D$, is a number close to zero (about 0.1). This would mean that the effect of a coarse particle is largely due to the fact that the blowing (burn rate) tends to be low, and not because there is a big particle sticking up into the wall layer. Also, the value of $N_u$ is close to 0.8; such a dependence is mindful of classical convective heating. The pressure dependence is about 0.2, and the rate dependence is about 0.5; several factors are involved in these dependencies,[9] and the results do not cause any single one to stand out.

## Response Function Components

With $E$ established in the model, a perturbation analysis similar to that performed by Renie and Osborn (but with a different combustion model and without the effect of heterogeneity proposed here)[15] provides the linear velocity-coupled response. An important quasisteady assumption made is that the properties of the wall layer ($E$ in this case) under an oscillatory flow are the same as during the equivalent steady-state condition. This assumption is not normally stated as such, the usual meaning being that the combustion properties of the gas phase follow steady-state behavior without time lags. That is really a second quasi-steady-state gas assumption, normally valid for the axial modes of a rocket motor. It is recognized that the wall layer assumption may not be valid, in particular for finite amplitude oscillations,[21] but it is retained for the present, pending better information.

The response function components are summarized as

$$R_c / n_p = 1/L \tag{4}$$

$$R_\alpha / n_\alpha = [1 - (K_p/n_\alpha)\sigma_p (T_{w_0} - T_i)(1 - 1/\lambda)]/L \tag{5}$$

$$R_u / n_u = 1/L \tag{6}$$

where

$$L = 1 - \sigma_p (T_{w_0} - T_i)(1 - 1/\lambda) + \sigma_T (\lambda - 1) \tag{7}$$

$$n_u = \left( \frac{\partial \ell n m}{\partial u / a_0} \right)_{p,\alpha,T_i} \tag{8}$$

The definition of velocity exponent, Eq. (8), corresponds to the definition of the response function, $R_u$, as required in current motor stability analysis to represent the driving.[22-24] The other terms have been defined previously.[1] The real parts of Eqs. (4) and (5) are of interest for describing the pressure-coupled driving.[1] The imaginary parts of Eqs. (5) and (6) are of interest for the velocity-coupled driving.

## Parameteric Results

A series of computations were performed over ranges of the controlling combustion parameters, as reported previously,[1] but now the mean cross-flow velocity was added as a parameter and the velocity-coupled response parameters were added as calculated dependent variables. Results at zero cross flow, where the velocity-coupled response would be zero, were reported previously.[1] As noted then, results of this type are colored by the particular combustion model used; they should be limited to seeking and explaining major effects and trends for the stimulation of ideas and any insight or guidance they can provide. No claim is made about the prediction of response functions, and in all candor the method is not ready for that. The results with cross flow are summarized below.

## Effects of Controlling Ballistics Properties

Cross flow increases the burning rate, $r_0$, in accordance with the addition of Eq. (2) to the energy balance equations and in good agreement with the data of King.

In the absence of cross flow, the effects of pressure and particle size on pressure exponent, $n_p$, follow from the shifts in multiple flame structure, as described by this and several other combustion models of this type. Values of $n_p$ are of the order $10^{-1}$. Highest values occur at combinations of low pressure and fine particle size. Minima occur at an intermediate particle size that decreases with increasing pressure, or where diffusion control happens to be strongest. Cross flow tends to increase pressure exponent, usually by a small amount.

Cross flow tends to reduce temperature sensitivity, $\sigma_p$, which is an interesting result in itself and has been noted previously by others.[25,26] However, in terms of response function behavior, this trend is compensated by increased surface temperature $(T_{w_0} - T_i)$ and decreased $\sigma_T$ due to cross flow. The fact that these variables tend to change in these directions together mitigates the effect of $\sigma_p$ on combustion response,[1] and probably helps to keep instability behavior within bounds.

Whereas cross flow tends to increase $n_p$ by a small amount, it tends to decrease the concentration exponent, $n_\alpha$, by a small amount. Since values of $n_\alpha$ are of the order $10^0$-$10^1$, as reported previously,[1] this effect of cross flow is potentially the dominant effect on pressure-coupled driving.

The velocity exponent $n_u$ is a very interesting parameter. Calculated values span three orders of magnitude, ranging from the order $10^{-2}$ to the order $10^1$. The values follow a complicated behavior pattern which, curiously, parallels those of $n_\alpha$[1] and is opposite to those of $n_p$ in many respects. This behavior is illustrated by Table 1. Lowest values occur at combinations of low pressure and fine AP size, which correspond to conditions for highest values of $n_p$. Larger, although not necessarily largest, values occur at combinations of high pressure and coarse AP. These also occur with $n_\alpha$,[1] and correspond to secondary reductions in $n_p$.[17] A second group of larger, which are sometimes largest, values of $n_u$ occur at an intermediate particle size that decreases with increasing pressure. These also occur with $n_\alpha$, and correspond to the pressure exponent minima. There are also secondary minima in $n_u$ and $n_\alpha$, which occur at a coarser intermediate particle size that decreases with increasing pressure and which correspond to secondary maxima in $n_p$. These patterns undoubtedly arise from the shifts in the multiple flame structure, but the reason why there should be parallel and opposite behavior among the three exponents has not been ascertained.

The effect of AP content on $n_u$ is much stronger and more consistent than it is upon $n_\alpha$. Similarities with $n_\alpha$ would come from the flame structure effects, but there are two additional factors not contained within $n_\alpha$. One factor affecting the increase with AP content is the increase in $a_0$. The other, and larger, effect is the fact that the threshold velocity becomes closer to the given cross flow as AP content increases. Stated another way, once the threshold is exceeded the effect of increasing cross flow is to reduce $n_u$ substantially. This effect of cross flow on $n_u$ is much larger than that on $n_\alpha$. Stated yet another way, the largest erosive burning effect occurs when the threshold is first exceeded, with diminishing returns thereafter.

The effects of AP particle size on the controlling ballistics properties[1] do not qualitatively change with cross flow.

## Effects on Pressure-Coupled Response

Calculated peak values of the real part of $R_c$ were reported previously.[1] Values ranged from the order $10^{-1}$ to the order $10^0$. Cross flow has a small effect on these numbers, and does not change the qualitative influence of particle size. Usually, the peak value increases with cross flow, but the effect is not systematic.

Calculated peak values of the real part of $R_\alpha$ were also reported previously.[1] Values ranged from the order $10^0$ to the order $10^1$ (except for negative values on the oxidizer-rich side of stoichiometry), an order of magnitude larger than the values for $R_c$. Cross flow generally reduces these values. Usually the effect is small, but it is sometimes substantial where the values are relatively large in the absence of cross flow. The reductions tend to be smaller at higher AP content. The qualitative influence of AP particle size is not changed by cross flow.

Since values of $R_\alpha$ are much larger than $R_c$, the net effect of cross flow would be to reduce the peak values of pressure-coupled driving. Perhaps a more significant effect is the shift of the peak response frequencies to higher values, due to the increased burning rate. This will tend to be stabilizing to the pressure-coupled driving at lower frequencies, and destabilizing at higher frequencies.

**Table 1 Calculated values of the velocity exponent (Cohen-Strand model), $U_0 = 200$ m/s**

| | $D$, μm | | | | |
|---|---|---|---|---|---|
| $P$, MPa | 5 | 20 | 90 | 200 | 400 |
| | | $\alpha_0 = 0.73$ | | | |
| 1.4 | 0.04 | 2.4 | 1.7 | 2.1 | 4.4 |
| 3.4 | 1.6 | 1.4 | 2.6 | 4.1 | 7.5 |
| 6.8 | 3.3 | 1.9 | 4.3 | 7.5 | 11.8 |
| 13.6 | 1.2 | 2.7 | 7.6 | 12.2 | 14.7 |
| | | $\alpha_0 = 0.80$ | | | |
| 1.4 | 0.07 | 4.2 | 3.1 | 1.6 | 3.8 |
| 3.4 | 2.2 | 9.5 | 3.3 | 3.5 | 6.7 |
| 6.8 | 5.6 | 3.4 | 4.5 | 6.7 | 13.0 |
| 13.6 | 9.6 | 3.2 | 5.5 | 13.4 | 21.4 |
| | | $\alpha_0 = 0.88$ | | | |
| 1.4 | 2.5 | 17.4 | 59.2 | 24.2 | 12.2 |
| 3.4 | 10.7 | 42.6 | 27.1 | 18.4 | 11.8 |
| 6.8 | 23.6 | 65.7 | 19.2 | 16.3 | 14.9 |
| 13.6 | 44.7 | 29.2 | 11.9 | 20.2 | 32.4 |

## Effects on Velocity-Coupled Response

The typical appearance of the imaginary part of $R_u$ as a function of dimensionless frequency is shown in Fig. 1. There are two regions of interest: at low frequency, burn rate fluctuations are phase-leading and the imaginary part is positive; at high frequency, burn rate fluctuations are phase-lagging and the imaginary part is negative. This distinction is very important to the design of a rocket motor because it determines (in theory) whether velocity-coupling is a stabilizing or destablizing effect.[27] Three key frequencies are noted: that corresponding to the peak positive response ($\Omega_P$), that corresponding to the peak negative response ($\Omega_N$), and that corresponding to the sign change ($\Omega^*$). It is a typical result that the negative values reach somewhat larger absolute magnitudes than the positive values, and that the negative part exhibits a relatively broad and flat peak region compared to the positive part.

Qualitative effects of the temperature-sensitivity parameters $\sigma_p$ and $\sigma_T$ upon the velocity response are the same as upon the pressure response.[1] Increasing $\sigma_p$ tends to increase peak values and narrow or sharpen peak regions. Increasing $\sigma_T$ tends to decrease peak values and narrow peak regions. As mentioned earlier, these two parameters tend to vary in the same direction, such that there is a compensating effect on the peak response magnitudes. The peak response magnitudes are proportional to the velocity exponent.

The imaginary part of the compositional response, $R_\alpha^{(i)}$, exhibits qualitatively similar features and properties as the im-

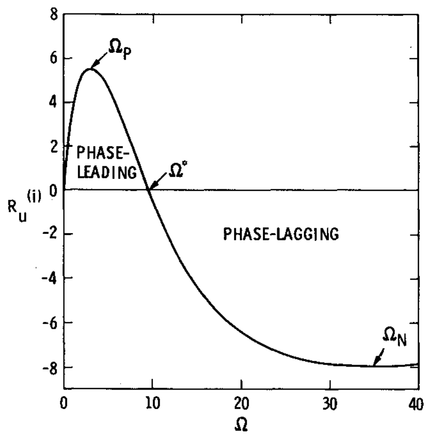


**Fig. 1 Typical velocity response function curve, 73% AP/HTPB, 200-μm particle size, 6.8-MPa pressure, 200-m/s cross flow.**

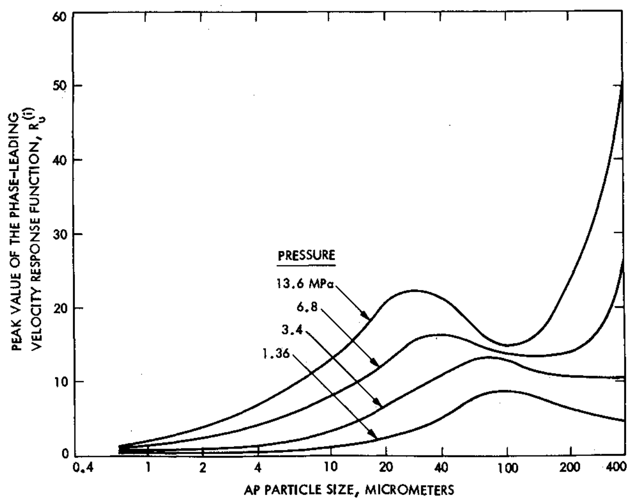


**Fig. 2 Effect of AP particle size and pressure on peak values of phase-leading (positive) velocity response, 88% AP/HTPB, 200-m/s cross flow.**

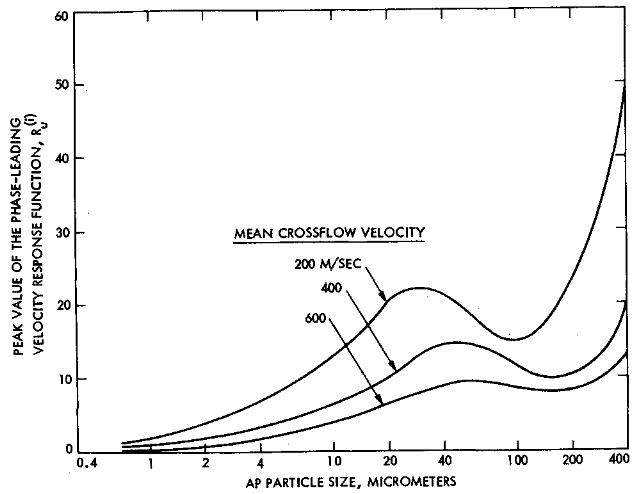


**Fig. 3 Effect of AP particle size and mean cross-flow speed on peak values of phase leading (positive) velocity response, 88% AP/HTPB, 13.6-MPa pressure.**

aginary part of the velocity response. This was also true in comparing the real parts of the compositional and pressure response, as reported previously.[1] However, whereas the compositional response always achieved magnitudes far larger than the pressure response, such that it could be inferred that the heterogeneity would dominate pressure-coupled driving, the magnitudes of the compositional and velocity responses are more comparable. The magnitudes of the compositional response tend to exceed the velocity response at lower AP concentrations, lower pressure, and with finer AP particle sizes. The magnitudes of the velocity response tend to exceed the compositional response at higher AP concentrations, higher pressure, and with coarser AP particle sizes. Therefore, with regard to velocity-coupling, the heterogeneity is important, although not as dominating an influence as with pressure coupling.

Another consequence of relatively large values of both the compositional and velocity response is that the velocity-coupled response can tend to have larger values than the pressure-coupled response. Experimental measurements or deductions of absolute values in excess of 10 have been frequent and are often viewed with suspicion. This work supports such large values, and does so with confidence because theoretical and experimental values of the exponents (for pressure, $n_p$, for composition, $n_\alpha$, and for velocity, $n_u$) are in fairly good agreement, and because values of $n_u$ are generally much larger than values of $n_p$.

Figure 2 shows the effects of AP particle size and pressure upon the phase-leading peak values of the imaginary part of the velocity response. These calculations were made for 88% AP and a mean flow velocity of 200 m/s, but the results typify the qualitative behavior. The most striking feature is the sharp increase with pressure in the coarse AP regime, and the very large peak values attainable. In general, the peak response values increase with pressure and particle size, although an intermediate maximum appears at an intermediate particle size. Figure 3 shows the effects of AP particle size and cross-flow velocity at constant pressure. The striking feature here is the sharp increase with decreasing velocity in the coarse AP regime. In general, the peak response values increase with decreasing velocity over the range of velocities considered. Effects upon the phase lagging peak and upon the analogous peaks for $R_\alpha^{(i)}$ are qualitatively the same. Quantitatively, the effects of pressure, particle size, and velocity on the peak $R_\alpha^{(i)}$ are not as large as appear in Figs. 2 and 3.

The three key frequencies tend to increase with variables that bring about an increase in burning rate: increased pressure, velocity and AP content, and decreased particle size. Whether this tends to be helpful or harmful at a particular frequency will, in theory, depend upon the distribution of propellant burn surface relative to the acoustic wave.[27]

Perhaps the key finding is that the velocity-coupled response is largest at combinations of coarse AP/low burn rate, high pressure, and low mean cross-flow velocity (but in excess of threshold velocity). This is gratifying in that it is fully consistent with experience on triggered instabilities, assuming that linear velocity coupling is related to that problem. Low velocity appears to be important because the erosive response becomes proportionately largest as the threshold is approached. This seems paradoxical because the phrase "velocity-coupling" would imply that more velocity is needed to make it stronger. What is important is the effect of the velocity perturbation, rather than velocity itself. The importance of low burn rate is related to the fact that the blowing is reduced, such that the transport of energy through the wall layer due to the velocity perturbations is facilitated. Coarse AP tends to produce low burn rate, but there are competing effects of high pressure with regard to the heat feedback augmentation. In the framework of the combustion model used, there is a more important effect of coarse AP and high pressure that is in operation here. These promote combustion control by the AP monopropellant flame, which is a relatively low energy and kinetically limited flame. From a fundamental standpoint, such a flame is more sensitive to perturbations imposed. Figures 2 and 3 would suggest that 400-$\mu$m AP be avoided in situations where problems with velocity-coupling would be anticipated, or as a possible remedy.

Other figures of this type could be generated to furnish developmental guidelines, but it is cautioned that specific results are subject to uncertainties in the combustion model and that more work is needed before the elements can be combined to properly apply this theoretical approach.[1]

## Conclusions

The effect of cross flow on burning rate can be described by adding a term of the form of Saderholm's law to the energy balance equations contained within a combustion model. On this basis, it is deduced that the effect is being described by a convective heating mechanism, and that the role of AP particle size is largely its influence upon blowing rather than surface roughness.

Cross flow tends to increase the pressure exponent, and to reduce the concentration exponent and temperature sensitivity; it does not change the effects of AP particle size on these parameters. After the threshold velocity is exceeded, further increases in cross flow reduce the sensitivity of burn rate to cross flow (velocity exponent). The effect of AP particle size on the velocity exponent depends upon AP content and pressure, and closely parallels its effect on the concentration exponent. It is an interesting result that there is such parallel behavior, and that it is opposite to the pressure exponent behavior.

Cross flow generally has a small effect on the magnitudes of the elements of pressure-coupled driving, and does not change the effects of AP particle size. The compositional response element continues to be much larger than the classical pressure-response element. Cross flow does, however, shift peak response frequencies to higher values.

Largest peak values of the elements of velocity-coupled driving occur with coarser AP particles at lower peak response frequencies and at higher pressures. The effects of cross flow are to reduce the peak magnitudes (as cross flow increases away from threshold velocities), in particular for the classical velocity-response element, and to increase the key response frequencies. The compositional response element and the velocity response element have comparable magnitudes, and it is interesting that they are much larger than the pressure response element. This conclusion is supported by experimental data for the respective exponents.

Although more theoretical and experimental work is required to substantiate the mechanism for combining the various response elements, significant insight has been provided by examining their individual properties. Results are consistent with experimental trends regarding triggered instability.

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